Preface to Circle 1



Mathematics seems to endow one with something like a new sense.

Charles Darwin, originator of the theory of evolution (1809–1889)



Who cares for math!

There are four great sciences ... Of these sciences the gate and key is mathematics, which the saints discovered at the beginning of the world.

Roger Bacon, philosopher and scientist (1214–1292)

The good Christian should beware of mathematicians and all those who make empty prophecies. The danger already exists that mathematicians have made a covenant with the devil to darken the spirit and confine man in the bonds of Hell.

Saint Augustine, theologian and philosopher (354–430)

It is not a story of brilliant achievement, heroic deeds, or noble sacrifice. It is a story of blind stumbling and chance discovery, of groping in the dark and refusing to admit the light. It is a story replete with obscurantism and prejudice, of sound judgment often eclipsed by loyalty to tradition, and of reason long-held subservient to custom. In short, it is a human story.

Tobias Dantzig, about the development of the notion of number, in *Number: The Language of Science*, Macmillan, 1930.

Mathematics is neither God's nor the devil's, but rather a human activity. Humans have created it in order to manage the world more easily, just as they invented the compass and the computer. Indeed, it can easily encourage emotions, as evidenced by the quotations above. But is it not the same with football or music? While some glorify mathematics and attribute divine qualities to it, others are horrified by it and think that it is necessary to have a special gift in order to understand it. All these are simply myths. Since mathematics is a human creation, it can be understood, practised and used. Everyone can learn how to play a guitar well enough to play simpler melodies and enjoy composing their own. It is the same with mathematics. You can understand it without problem, acquire it, and use it. How far you will go will depend only on your reasons for studying it and how interesting you find it.

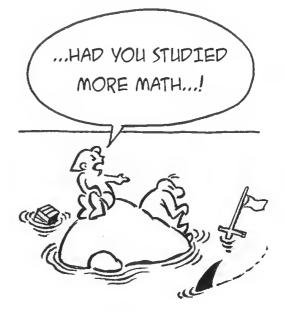
How much you need mathematics depends on what you do. In everyday situations it is almost unnecessary. Sometimes you need to add up, subtract, multiply, or divide something — usually natural or decimal numbers. Have you ever had — in an everyday situation — to multiply two fractions or calculate the circumference of a circle? However, everyday life is also becoming more complex. You receive more and more information and it is very important that you can distinguish truth from a lie and the good intentions from

attempts at manipulating. In the years to come it seems that this skill will be even essential for the survival of Humanity. Hannah Arendt, in her book *The Origins of Totalitarianism*, writes: "The ideal subject of totalitarian rule is not the convinced Nazi or the convinced Communist, but people for whom the distinction between fact and fiction (i.e. the reality of experience) and the distinction between true and false (i.e. the standards of thought) no longer exist." And this is where thinking and knowledge help a lot. *Mathematics is their important part*.



No matter what you do in your life, it is almost certain that there are situations in which mathematics will come in handy, since it is embedded in almost every human product, whether the mobile phone or knowledge about the nuclear processes occurring on the Sun. Maybe it is mathematics that you studied at school, maybe it is mathematics that you will find in specialised books, or maybe you will have to create it yourself. The main purpose of math educa-





tion is not so much to provide you with ready-made mathematical thoughts as to make you ready to think mathematically.

There is one more reason why (some) people deal with mathematics, and that is beauty. *Independent of any benefit, they are attracted by the harmony of mathematical ideas*. However, this beauty is often related to the applicability of mathematics. If the world seems harmonious to us, then the mathematics that will describe it needs to be just the same. The quest for beauty has more than once delivered great discoveries. As an example one can take the physicist and mathematician James Clerk Maxwell (1831–1879). Studying the already-known laws of electromagnetism, it seemed to him that they would be much "nicer" if he changed one equation "a little". Upon doing this, he obtained the laws on whose basis he concluded something completely unexpected – that there exist electromagnetic waves, and that the light itself is an electromagnetic wave. This discovery is the foundation of advanced information transfer from television to internet.



(The text is from the poem *Notturno* by the great Croatian poet Tin Ujević)

Before we start our joint journey, let us explode another myth: the myth about mathematics as unerring and perfect knowledge. The math we find in books is usually a finished product where everything is nicely arranged, as in a store of porcelain figures. Scared of breaking something, you dare not move. However, mathematics in its emergence is completely different. It is full of trial and error, dead ends, and misconceptions. Simply, it is alive. Only in the end all the failures are pushed "under the carpet", and only successes are shown. Everyone knows about Isaac Newton (1642–1727), the giant of thinking who laid the foundations of modern science. However, only a few know that he considered the discovery of the date of the world's origin as one of his most important discoveries. He calculated that God created the World in 3500 BCE. He made a mistake by only some twenty billion years, if not even more. Learning mathematics is actually the emergence of mathematics in your head. So, do not be afraid of imagination, trials and errors. You will learn math better with them than without them.



We will start our journey with numbers. Numbers are the oldest mathematical idea, but still also the most important one. Not only will we go through the basics of numbers in a way that will give you the confidence to really understand numbers and really know how to apply them, but you will also learn all the essential elements of mathematics through the example of the world of numbers. The example of numbers will be used to illustrate

what mathematical objects are and how they are applied, and what mathematical tools we use in their description and application. These elements are summarised in the *Calm Epilogue* of the book.

Humanity needed millennia to develop the world of numbers and methods for their description and application. While growing up you are expected to pass through this history briefly in a dozen years of education. On the basis of the experience of the whole of human civilisation and your education, we are now in the position to acquire, in several weeks, knowledge about numbers at a more mature level.