Preface to the Circles Series

Learning is much more similar to biological growth than to manufacture, where component parts are first produced, then fitted together.

W. Servais, T. Varga: *Teaching School Mathematics,*
A UNESCO Source Book, 1971.

During the twentieth century, mathematics experienced a real explosion of content and methods. Its modern core, however, consists of only a few principles. Acquiring these principles, understanding them, and learning the art of combining them in modelling more complex situations forms the mathematical minimum that I would like to present in these books (the circles). My goal is to present these principles in a simple – yet not simplified – manner, so that the reader actually acquires them, and does not just have the impression that she or he has acquired them. My goal is to present mathematics in a way that helps the reader to develop the mathematical dimension of his (her) humanity and does not just show him (her) the human dimension of mathematics. All the books present the same principles of mathematics, but at different levels of abstraction and complexity. The basic philosophy of the Circles Series is that *the acquisition of mathematics is not like constructing a building brick by brick, but more like the growth of a living organism in which the same principles that are present in the very beginning develop in each phase of growth.* These books have been conceived so that you can study them one after the other until you achieve a level of understanding with which you will be satisfied: a level that will allow you to acquire the appropriate mathematical dimension for your personal activities. You do not even have to start with the first book, but rather with the book you consider to be the right starting point to fill the gaps in your mathematical knowledge. The books are designed for self-study, and each book can be studied independently of the others. They cover the range from elementary mathematics to the mathematical foundation for any non-mathematical profession. The first book assumes that there has

already been a "zeroth book" – that the reader have acquired certain calculating skills and a certain dissatisfaction with her (his) understanding of what she (he) have been calculating. The first two books are intended primarily for teenagers. They enable them to gain confidence in their mastery of numbers, to gain a broader understanding of mathematics through the example of numbers and, eventually, to get rid of the frustrations that make it difficult for them to learn mathematics. This does not mean that one of the older readers cannot go back to their teenage years and make up for the missed mathematical knowledge.

Furthermore, each book is not a world in itself. Each one contains links to its own web page, `https://understandingmath.academy/math-circles/`, where auxiliary content can be found, as well as to `https://en.wikipedia.org/wiki/Portal:Mathematics` which is truly an enormous source of high-quality mathematical knowledge. If you have additional interest in particular topics, the links will guide you to the appropriate web pages.

Exercises in mathematics play the same role as practical work in mastering practical activities. Without them, it would be impossible to acquire the ability to actually do mathematics. You can know by heart the entire theory of processing wood, but if you do not invest enough time in physically working with wood, you will certainly never become a good carpenter. The problems accompanying each book let you check whether you have correctly acquired both the theory and the skill of its application. Therefore, each chapter is followed by problems and their solutions. They have been classified according to the topics of the chapter, so that when you have studied a certain topic, you can immediately solve the corresponding problems at the end of the chapter.

Although the focus in these books is on comprehension rather than calculation, calculating is an essential component of mathematics. Acquiring mathematical principles also requires adequate elementary calculation skills. Sooner or later, the need for more complex calculations may come along. Fortunately, this is where computers can be of significant help nowadays. An important part of acquiring core mathematical skills is developing the skill of using the computer in mathematical activities. On the one hand, we have powerful mathematics software which require time to learn how to use and are not free of charge. On the other hand, there are a number of simple, free math software, but each of them solves only a certain type of problem. Fortunately, in recent decades a simple and advanced free open-source software called *SageMath* has been developed. *SageMath* is essentially an elegant amalgamation of all the previous big mathematical

open-source software and has become a serious competitor to professional mathematics software. We will use *SageMath*'s simplest mode – when we need it, we will just visit the web page https://sagecell.sagemath.org/ and key in the appropriate commands in the marked space. The *SageMath* tutorial and the *SageMath* examples for each book can be found on the web page https://understandingmath.academy/sagemath-materials/.

We will start studying math together with two different students, an *Astonished* guy and an *Uninterested* one. Each of them has one dominant psychological characteristic in the learning process, both of which are present in ourselves in different ratios. My teaching will be supplemented by the *Professor*. He has all the characteristics of a good teacher that I lack.
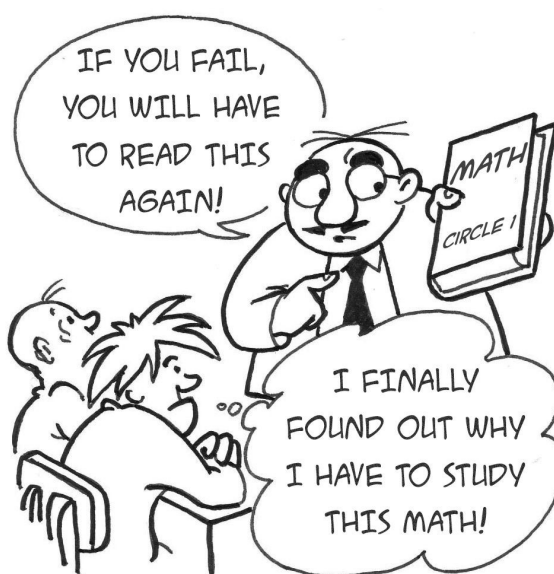
When we were nearing the end of compiling material for the books, my cartoonist Filipović and I felt that we were missing something important. We finally realised we were minus a female character! What a subconscious male embarrassment! We imagined that she would be a critical and direct down-to-earth punk girl to whom you can't sell everything, and the cartoonist had already started to design her character. Unfortunately, my dear Filipović died and our *DowntoEarth* girl was never realized. I imagine that she is still here, so I ask you to imagine her presence while reading these books. They will definitely be better that way.

Many examples in this book are related to my own life and the environments I have lived in, mainly in Croatia and in Bosnia and Herzegovina. Whether you are from New York or from Madagascar, this does not change a thing. Our concrete lives are the best way to share universal human values.

A brief description of each book's contents can be found below:

UNDERSTANDING AND DOING MATH – CIRCLE 1: What are Math Objects? Illustrated with Numbers. The familiar example of numbers is used to illustrate what mathematical objects are, how we manage them, and how we apply them.

UNDERSTANDING AND DOING MATH – CIRCLE 2: What are Math Tools? Illustrated with Numbers. At a higher level of precision than in Circle 1, various mathematical tools are described, as well as how they are applied in modelling more complex situations.

UNDERSTANDING AND DOING MATH – CIRCLE 3: Mathematics of Geometry and Geometry of Mathematics The goal of this Circle, in addition to the acquisition of geometric knowledge and related mathematics, is primarily for the reader to acquire geometric ideas and their mathematical realization, and to prepare him (her) for learning the geometry of mathematics: how geometric ideas put light into the most abstract parts of mathematics and thus make them more efficient.

UNDERSTANDING AND DOING MATH – CIRCLE 4: Mathematics: From Ideas to Realizations. In addition to acquiring two important and mathematically different parts of mathematics (continuous mathematics of change versus discrete mathematics of graphs), the main purpose of this part is to show the reader, using these examples, how to go from an idea to its mathematical realization.

UNDERSTANDING AND DOING MATH – CIRCLE 5: Mathematical Ideas and Structures. At a higher level of precision than in the previous books, basic mathematical ideas are analysed and mathematical structures for the realisation of these ideas are developed. These structures, together with methods for their combination and application, form the core of modern mathematics.

UNDERSTANDING AND DOING MATH – CIRCLE 6: The Architecture of Mathematics. At a level of precision characteristic of advanced mathematics, the logical structure of modern mathematics, as a distinctive and integral area of human activity, is presented.